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A NOVEL SUPERPOLYCONVOLUTION OPERATOR AND APPLICATIONS

Minh Khoa Nguyen* and Le Trong Phuoc Bui

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We investigate a superpolyconvolution operator associated with the Fourier sine integral transform and characterized by the weight function $\gamma(y_1) = \sin(y_1)$. The operator generalizes the classical polyconvolution framework by incorporating a five-fold transform decomposition that extends the traditional four-transform structure. Within this analytic setting, we establish several fundamental results, including algebraic properties, L_1-integrability, a Parseval-type identity, and Young-type inequalities in mixed L_p spaces, together with a Titchmarsh-type uniqueness theorem ensuring the distinctness of the associated functions. Applications are also discussed, showing that the proposed operator allows explicit solutions to various integral equations and integro-differential equation of Barbashin type.

1. Introduction

The classical convolution operator for integral transforms were studied at the beginning of 20th century, at first the convolutions operators for the Fourier integral transform, Fourier cosine integral transform, the Laplace integral transform, the Mellin integral transform and after that the convolution operators for the Hilbert integral transform, the Hankel integral transform, the Stieltjes integral transform. In 1967, V. A. Kakichev [9] proposed a constructive method for defining the convolution operator with the weight function which is more general than the convolution. In 1998, V. A. Kakichev and

* *Corresponding Author*

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Minh Khoa Nguyen
Department of Mathematics
Electric Power University
235 Hoang Quoc Viet, Nghia Do, Ha Noi
Vietnam.

E-mail: khoanm@epu.edu.vn

Le Trong Phuoc Bui
Department of Mathematics
Electric Power University
235 Hoang Quoc Viet, Nghia Do, Ha Noi
Vietnam.

E-mail: phuocblt@epu.edu.vn