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WEAK CONVERGENCE OF DOUBLE SEQUENCES AND ITS GEOMETRIC PROPERTIES

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In this article, we investigate weak convergence and weak Cauchy of double sequences in a normed linear space. We introduce the classes of weakly convergent, weakly null, and weakly bounded double sequences. We examine their linear structure, completeness and inclusion relations, and further explore the geometric properties of these three classes.

1. Introduction

A double sequence in a normed linear space X is a mapping

$$x : \mathbb{N} \times \mathbb{N} \longrightarrow X,$$

where $x = x_{nk}$ for each $(n, k) \in \mathbb{N} \times \mathbb{N}$. Thus, a double sequence is denoted by $(x_{nk})_{n,k\in\mathbb{N}}$ and can be represented as a double infinite array of elements of X.

Throughout the article, $\mathbb{N}$, $\mathbb{R}$, $\mathbb{K}, c, c_0$, ℓ_∞, ℓ_p, and $\mathbb{R}_{\geq 0}$ denote the set of natural numbers, the set of real numbers, the scalar field, the class of convergent sequences, the class of null sequences, the class of bounded sequences, the class of p-absolute summable sequences and the set of non-negative real numbers, respectively.

In 1900, Pringsheim [9] introduced the notion of convergence for double sequences, now commonly referred to as convergence in the Pringsheim's sense. Bromwich [4] contains some of the earlier studies on double sequence spaces. Móricz [8] extended the spaces c and c_0

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