# Indian Journal of Mathematics Dharma Prakash Gupta Memorial Volume

Volume 60, No. 2, 2018

#### Lead Guest Editors:

(1) Dr. Mourad E. H. Ismail, Professor, Department of Mathematics, University of South Florida, 4202 East Fowler Avenue, Phy 114, Tampa Florida, U.S.A., E-mail: mourad.eh.ismail@gmail.com

(2) Dr. Mona Khare, Professor, Department of Mathematics, University of Allahabad, Allahabad 211 001, India, E-mail: ams10marg@gmail.com

## CONTENTS

### Utkal Keshari Dutta, Sushree Sangeeta Pradhan and Prasanta Kumar Ray

REGULARIZED PRODUCTS OVER BALANCING AND LUCAS-BALANCING NUMBERS

171 - 179

181-206

**Abstract:** It is well known that using zeta-function regularization, the regularized products over all positive integers and over all primes can be calculated. In the present study, we evaluate the regularized products over balancing and Lucas-balancing numbers using the notion of zeta-function regularization.

#### Ashwini Kulkarni and Sarita Thakar

INTEGRABILITY ANALYSIS OF GENERALIZED MODIFIED EMDEN TYPE EQUATION

**Abstract:** In this paper, we have defined Lagrangian function in rational form which completely describes generalized modified Emden type equation. First integrals of generalized modified Emden type equation are obtained with the help of Noether's theorem. These first integrals are integrated by using Lie point symmetries.

#### H. Kiskinov and A. Zahariev

ON ABSTRACT INTEGRAL EQUATIONS WITH TWO NONLINEAR VOLTERRA TYPE OPERATORS IN METRIC SPACES 207-234

**Abstract:** In the present work a class of abstract integral equations of second kind with two nonlinear Volterra type operators is considered. Sufficient conditions for the existence and uniqueness of the solutions of this class integral equations are obtained. The corresponding integral inequalities are studied too and some applications of the obtained results to integral inequalities involving maxima are given.

#### Rajib Mandal

ON SOME UNIQUENESS RESULTS FOR CERTAIN TYPE OF DIFFERENTIAL-DIFFERENCE POLYNOMIALS

**Abstract:** In this paper, we have focused on uniqueness results when certain types of differential-difference polynomials of finite order share a small function under relaxed sharing hypotheses. The results improve a number of existing results and rectify some gaps and errors in [21].

#### G. S. Srivastava and Chhaya Singhal

On the q-order and q-type of analytic matrix functions in complete Reinhardt domain

271 - 286

301-310

235-269

Abstract: In this paper, we study the growth of analytic matrix functions and introduce their (q)-order and (q)-type. We also obtain the coefficient characterizations for these growth parameters, independently of the scalar analytic function of two complex variables associated with it.

#### Vladimir Samodivkin

MINIMUM ROMAN DOMINATING FUNCTIONS: ADJACENCY 287-300

**Abstract:** A Roman dominating function (RD-function) on a graph G = (V(G), E(G))is a labeling  $f : V(G) \longrightarrow \{0, 1, 2\}$  such that every vertex with label 0 has a neighbor with label 2. The weight f(V(G)) of a RD-function f on G is the value  $\sum_{v \in V(G)} f(v)$ . The Roman domination number  $\gamma_R(G)$  of G is the minimum weight of a RD-function on G.

The  $\gamma_R$ -graph of a graph G, is any graph which vertex set is the collection  $\mathscr{D}_R(G)$  of all minimum weight RD-functions on G. We define adjacency between any two elements of  $\mathscr{D}_R(G)$  in several ways, and initiate the study of the obtained  $\gamma_R$ -graphs.

#### Talal Ali Al-Hawary

MATROID'S FILTERBASE

**Abstract:** In this paper, we define the notion of a filter in a matroid and analogous to the idea of a base for a matroid, we introduce the idea of a filterbase. We study properties of matroid filterbases, gathering points and convergence of filterbases. We characterize bimatroids and the closure operation of a matroid in terms of filterbases.

#### Vijayakumar S. Muni, Venkatesan Govindaraj and Raju K. George

Controllability of fractional order semilinear systems with a delay in control 311-335

**Abstract:** In this article, we study the controllability of finite-dimensional dynamical control systems modeled by fractional order  $\alpha \in (0, 1)$  semilinear autonomous differential equations with a constant time delay in control function. Initially, we derive a necessary and sufficient condition for the controllability of the corresponding linear fractional order delay system. Then under the assumption of boundedness of nonlinearities, we prove

that the actual system is also controllable by employing Schauder fixed point theorem. An example is given to illustrate the theoretical results.

#### B. Ungor, H. Kose, Y. Kurtulmaz and A. Harmanci

A NIL APPROACH TO SYMMETRICITY OF RINGS

337-357

Abstract: We introduce a weakly symmetric ring which is a generalization of a symmetric ring and a strengthening of both a GWS ring and a weakly reversible ring, and investigate properties of the class of this kind of rings. A ring R is called *weakly symmetric* if for any  $a, b, c \in R$ , abc being nilpotent implies that Racrb is a nil left ideal of R for each  $r \in R$ . Examples are given to show that weakly symmetric rings need to be neither semicommutative nor symmetric. It is proved that the class of weakly symmetric rings lies also between those of 2-primal rings and directly finite rings. We show that for a nil ideal I of a ring R, R is weakly symmetric if and only if R/I is weakly symmetric. If R[x] is weakly symmetric, then R is weakly symmetric, and R[x] is weakly symmetric ring which satisfies Köthe's conjecture is exactly an NI ring. We also deal with some extensions of weakly symmetric rings such as a Nagata extension, a Dorroh extension.

\*\*\*\*\*\*\*