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CONTENTS WITH ABSTRACT

K. Suchithra, B. Adolf Stephen, A. Gangadharan and S. Sivasubramanian

Fekete-Szegö inequality for certain subclass of analytic functions related to complex order 453-461

Abstract: In the present investigation, the authors obtain Fekete-Szegö inequality for certain normalized analytic function f(z) defined on the open unit disk for which $1 + \frac{1}{b} \left[\frac{\alpha\beta z^3 f'''(z)+(2\alpha\beta+\alpha-\beta)z^2 f''(z)+zf'(z)}{\alpha\beta z^2 f''(z)+(\alpha-\beta)zf'(z)+(1-\alpha+\beta)f(z)}-1\right]$ ($0 \le \beta \le \alpha \le 1, b \ne 0$, a complex number) lies in a region starlike with respect to 1 and is symmetric with respect to the real axis. Also certain applications of the main result for a class of functions of complex order defined by convolution are given. As a special case of our result, we obtain Fekete-Szegö inequality for a class of functions defined through fractional derivatives.

H. W. Gould and Jocelyn Quaintance

OPERATORS, INVERSE SERIES RELATIONS AND OTHER EXPANSIONS CONNECTED WITH GENERALIZED GOULD-HOPPER POLYNOMIALS 463-487

Abstract: In this paper, we extend the main results of [8] by replacing the function $y = e^{px^r}$ by the function $y = e^{g(x)}$, where g(x) is treated as an arbitrary formal power series in x. There are

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five parts to our paper. In Section 1, we generalize the Gould-Hopper Polynomial [8] $H_n^r(x, a, p)$ and its companion operator $\mathcal{D} \equiv D_x - prx^{r-1} + \frac{a}{x}$, where r, a, and p are arbitrary complex numbers, and n is a non-negative integer. In Section 2, we discuss an inversion theorem involving the generalized Gould-Hopper polynomials. In Section 3, we generalize the operational formulas of Carlitz [4] in terms of the companion operator associated with the generalized Gould-Hopper polynomials of Section 1. In Section 4, we analyze the function $G_n(x, g) = e^{g(D_x)}x^n$, which is dual to the generalized Gould-Hopper polynomials of Section 1. Finally, in Section 5, we discuss the properties of $G_n(x, g)$ associated with Appell nature of its derivative.

Halit Orhan and Elif Güneş

NEIGHBORHOODS AND PARTIAL SUMS OF ANALYTIC FUNCTIONS BASED ON GAUSSIAN HYPERGEOMETRIC FUNCTIONS 489-510

Abstract: By making use of the known concept of neighborhoods of analytic functions, we have proved several inclusion relations associated with the (n, δ) -neighborhoods of various certain subclasses of analytic functions of complex order, which are introduced here by means of the Linear operator theory. Special cases of some of these inclusion relations are shown to yield known results. Furthermore, we consider the partial sums of analytic functions.

Zhong-Quan He and Shih-Sen Chang

A system of nonlinear variational-like inclusions with η -monotone mappings in Hilbert spaces 511-522

Abstract: In this paper, we introduce and study a new system of nonlinear variational-like inclusions with η -monotone mappings

in Hilbert spaces. Using Banach contract mapping principle, we prove the solutions existence and uniqueness for the system of variational inclusions. we also construct a new algorithm for approximating the solutions of the system of variational inclusions and discuss the convergence of iterative sequences generated by the algorithm. The present results improve and extend many known results in the literature.

B. D. Acharya

Representing hypergraphs by families of sets with given properties

> Abstract: This paper reports my investigations on the problem of representing a hypergraph by a family of subsets of a nonempty set X that has a set of subfamilies each of which satisfies a given property [P], particularly when [P] is given to be the whole-intersection property. This has led to a generalization of the notion of an *intersection graph* and that of the *intersection* number of a graph to hypergraph theory. It is proved that every simple hypergraph is whole-intersection representable and hence an attainable upper bound on the *whole-intersection number* of a finite simple hypergraph is obtained. From the proof of this representation theorem, the idea of constellar hypergraph C(H)of a T_0 -hypergraph is motivated. The self-constellar hypergraphs (*i.e.*, hypergraphs H for which $C(H) \cong H$) are determined and it is shown that every simple hypergraph is *contractible* on to a self-constellar hypergraph. The notion of whole-intersection representation of a hypergraph is seen to enable one to derive various new hypergraphs from a given hypergraph. Especially, investigation on the line hypergraph $\mathcal{L}(H)$ appears to be promising.

B. A. Frasin and G. Murugusundaramoorthy

523-536

FRACTIONAL CALCULUS TO CERTAIN FAMILY OF ANALYTIC FUNCTIONS DEFINED BY CONVOLUTION 537-548

Abstract: In the present paper, we obtain distortion theorems using fractional calculus techniques for a class $ST(\Phi, \Psi, t, \gamma)$ which consists of analytic and univalent functions with negative coefficients.

J. Stella Irene Mary

Spectra of composition operators and their Aluthge transforms 549-555

Abstract: In this paper, the normal approximate point spectrum, approximate point spectrum of composition operators and that of their generalized Aluthge transformations as weighted composition operators have been characterized.

Somashekhar (Som) Naimpally

PROXIMAL AND OTHER CHARACTERIZATIONS OF UNIFORM INVARIANTS 557-565

Abstract: Every uniform space has uniform invariants such as completeness, total boundedness which are not proximal invariants. For example, every Efremovič proximity has a compatible totally bounded uniformity U*; so total boundedness cannot be characterized by an Efremovič proximity. A question arises: is it possible to characterize a uniform invariant in terms of proximities? In this paper, restricting to a metric space, we consider several compatible proximities and show that some of the well known uniform invariants can be characterized by pairs of proximities.

Li Yang and Shih-Sen Chang

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Algorithms for solving quasi-variational inclusion problem and fixed point problem 567-583

> Abstract: The purpose of this paper is by using a new iterative scheme for finding a common element of the set of solutions for the quasi-variational inclusion problem with multi-valued maximal monotone mapping and inverse-strongly monotone mapping and the set of fixed points of nonexpansive mapping in Hilbert space. Under suitable conditions, some strong convergence theorems for approximating to this common element are proved. The results presented in the paper extend and improve some recent results in Chang-Lee-Chan [Applied Math. Mech., 29:5(2008), 1-11], Iiduka and Takahashi [Nonlinear Anal. TMA, **61** (2005), 341-350], Takahashi and Toyoda [J. Optim. Theory Appl., **118**(2003), 417-428], Nadezhkina and Takahashi [J. Optim. Theory Appl., **128**(2006), 191-201], and Zeng and Yao [Taiwanese Journal of Mathematics, (2006)]*.

M. S. Mahadeva Naika, R. Y. Denis and K. S. Bairy

ON SOME RAMANUJAN-SELBERG CONTINUED FRACTION 585-596

Abstract: On page 55 of his 'lost' notebook, Ramanujan has recorded several P-Q eta-function identities with two moduli. In this paper, we establish several P-Q modular equations of degree 4. We also establish modular relations and explicit evaluations of Ramanujan-Selberg continued fraction.

B. D. Acharya, K. A. Germina, Kumar Abhishek,S. B. Rao and Thomas Zaslavsky

POINT- AND ARC-REACHING SETS OF VERTICES IN A DIGRAPH 597-609 Abstract: In a digraph $D = (X, \mathcal{U})$, not necessarily finite, an arc $(x, y) \in \mathcal{U}$ is reachable from a vertex u if there exists a directed walk W that originates from u and contains (x, y). A subset $S \subseteq X$ is an arc-reaching set of D if for every arc (x, y) there exists a diwalk W originating at a vertex $u \in S$ and containing (x, y). A minimal arc-reaching set is an arc-basis. S is a point-reaching set if for every vertex v there exists a diwalk W to v originating at a vertex $u \in S$. A minimal point-reaching set is a point-basis. We extend the results of Harary, Norman, and Cartwright on point-bases in finite digraphs to point- and arc-bases in infinite digraphs.

Mo. Rokibul Islam, M. Habibur Rahman and M. Ali Akbar

AN ANALYTICAL APPROXIMATE SOLUTION OF FOURTH ORDER MORE CRITICALLY DAMPED NONLINEAR SYSTEMS 611-626

Abstract: In this article, the Krylov-Bogoliubov-Mitropolskii (KBM) method has been extended for obtaining the solution of fourth order non-oscillatory nonlinear systems characterizing more critically damped systems. The results obtained by the presented method agree with those obtained by the numerical method nicely. An example is solved to illustrate and verify the method.

W. T. Sulaiman

Two new mappings in connection to Hadamard's inequality 627-636

> **Abstract:** In this paper we point out to two new inequalities of the Hadamard type on the lines of Dragomir [2]. We established our main properties including convexity, monotonicity and boundedness using a simple new technique in the proof.

G. R. Hiremath

New metrization theorems in terms of semi-open covers

Abstract: *M*-spaces in the sense of *K*. Morita and the Moore spaces are generalizations of metric spaces in different directions that are respectively generalized as wM-spaces in the sense of T. Ishii and semi-metric spaces. In this article, characterizations of wM-spaces, semimetric spaces, pseudometrizable spaces, and metrizables spaces in terms of semi-open covers are established. The article highlights the natural existence of the semi- $w\Delta$ spaces (introduced by the author in [6]), the wM-spaces, and the pseudometrizable spaces.

K. A. Germina and Jisha Elizabath Joy

TOPOGENIC GRAPHS: II. EMBEDDINGS

645 - 661

637-644

Abstract: A graph G, not necessarily finite, is called *topogenic* if there exists a nonempty 'ground set' X and an injective 'setassignment' $f: V(G) \longrightarrow 2^X$, such that (i) the 'induced edge function' $f^{\bigoplus}: E(G) \longrightarrow 2^X - \{\emptyset\}$ defined by $f^{\bigoplus}(uv) = f(u) \bigoplus f(v)$, $uv \in E$, where ' \bigoplus ' denotes the binary operation of taking symmetric difference of the subsets of X, is also injective, and (ii) $\tau_f =$ $f(V(G)) \cup f^{\bigoplus}(E(G))$ is a topology on X; such a set-assignment f, if it exists, is called a *topologizing set-indexer* of G. A study of topogenic graphs was initiated in [7]. In this paper, after recalling some selected previous results, we show that every graph G has a *topogenic host* H, in the sense that G is contained as an induced subgraph in H which is topogenic, implying thereby that there is no 'forbidden subgraph characterization' of topogenic graphs.

M. K. Aouf

BASIC PROPERTIES OF A CERTAIN SUBCLASS OF *p*-VALENT FUNCTIONS WITH NEGATIVE COEFFICIENTS 663-679

Abstract: The object of the present paper is to derive several interesting properties and characteristics of the class $F_p(\lambda, \alpha, j)$ consisting of certain *p*-valent functions with negative coefficients. Also modified Hadamard products of several functions belonging to the class $F_p(\lambda, \alpha, j)$ are studied here.

David H. Armitage and Ivor J. Maddox

A NONLINEAR ANALOGUE OF MERCER'S THEOREM 681-687

Abstract: A nonlinear transformation involving the strong Cesàro mean of a sequence is studied and it is shown that the order condition obtained on the transformed sequence is best possible.

Satya Deo and Veerendra Vikram Awasthi

An inverse system of nonempty objects with empty limit

689-697

Abstract: In this article we give an explicit example, in the category of sets, of an inverse system with nonempty sets and onto bonding maps such that its inverse limit is empty.