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**Surjit Singh Khurana**

WEAK COMPACTNESS OF MEASURES IN STRICT TOPOLOGIES 167-177

**Abstract:** Let  $X$  be a completely regular Hausdorff space,  $C_b(X)$  the space of all bounded, scalar-valued continuous functions on  $X$ . With strict topologies  $\beta_z$ ,  $z = t, \tau, \infty, \sigma$ , on  $C_b(X)$ , we take on  $M_z(X) = (C_b(X), \beta_z)'$  the strong topology in the duality  $\langle C_b(X), M_z(X) \rangle$  (this is same as norm topology). In this paper, we give some characterizations of relatively weakly compact subsets of  $M_z(X)$  in terms of uniform countable additivity, uniform inner regularity, and uniform convergence of some special nets and sequences.

**S. S. Dragomir**

SOME INEQUALITIES FOR RELATIVE OPERATOR ENTROPY 179-194

**Abstract:** In this paper, by the use of some recent refinements and reverses of Young's inequality, we obtain some inequalities for relative operator entropy.

**S. Ray and S. Ghosh**

RELATIONS BETWEEN SOME HIGHER ORDER GENERALIZED DERIVATIVES 195-209

**Abstract:** It is well-known that the Borel, Laplace and Riemann derivatives are generalizations of the Peano derivative. Here we study the Laplace Riemann derivative and Borel Riemann derivative. We prove that the Laplace Riemann derivative is more general than the Peano derivative, Laplace derivative and Riemann derivative. We also prove that the Borel Riemann derivative is more general than the Peano derivative, Borel derivative and Riemann derivative.

**Bruno de Malafosse**

ON THE (SSE) WITH OPERATOR  $(W_a^0)_\Delta + s_x = s_b$  AND  $(W_a)_\Delta + s_x^0 = s_b^0$  211-235

**Abstract:** Given a sequence  $z = (z_n)_{n \geq 1}$  of positive real numbers and a set  $E$  of complex sequences, we write  $E_z$  for the set of all sequences  $y = (y_n)_{n \geq 1}$  such that  $y/z = (y_n/z_n)_{n \geq 1} \in E$ . In particular,  $c_z$ , or  $s_z^{(c)}$  denotes the set of all sequences  $y$  such that  $y/z$  converges. We denote by  $W_a = (w_\infty)_a$  and  $W_a^0 = (w_0)_a$  the sets of all sequences  $y$  such that  $\sup_n (n^{-1} \sum_{k=1}^n |y_k|/a_k) < \infty$  and  $\lim_{n \rightarrow \infty} (n^{-1} \sum_{k=1}^n |y_k|/a_k) = 0$ , respectively. By  $\Delta$  we denote the operator of the first difference defined by  $\Delta_n y = y_n - y_{n-1}$  for all sequences  $y$  and all  $n \geq 1$  with the convention  $y_0 = 0$ . In this paper we recall the solvability of the (SSE)  $W_a^0 + F_x = F_b$ , where  $F$  is either  $\ell_\infty$ , or  $c$  and we solve the (SSE)  $W_a + s_x^0 = s_b^0$ . Then, we deal with the solvability of the (SSE) with operator defined by  $(W_a^0)_\Delta + s_x = s_b$ . Finally, we solve the (SSE)  $(W_a)_\Delta + s_x^0 = s_b^0$ .

**Elizabeth Reshma M. T. and T. P. Johnson**

A NOTE ON PROPERTIES INHERITED BY SINGLY GENERATED EXTENSION OF FRAMES

237-250

**Abstract:** Generating a new frame from an existing frame by adding a single element has been studied by many authors. The purpose of this paper is to explore the conditions that are to be imposed on a given frame so that its extension holds a particular property. Based on the detailed analysis of the corresponding problem in classical topology, we derive here their pointfree counterparts which are more apparent and concise. The properties that we consider here are separability, second countability, connectivity, metrizable, compactness, countably compactness and Lindelöfness. Also we obtain here a unified condition for the maximality of the properties compactness, countably compactness and Lindelöfness relative to Boolean frames.

**Pierpaolo Natalini, Gabriella Bretti and Paolo Emilio Ricci**

ADJOINT HERMITE AND BERNOULLI POLYNOMIALS

251-264

**Abstract:** In recent papers, new sets of Sheffer and Brenke polynomials based on higher order Bell numbers have been studied, and several integer sequences related to them have been introduced. In this article other types of Sheffer polynomials are considered, by introducing a sort of adjointness property. As first examples, the adjoint Hermite and Bernoulli of the second kind polynomials are presented.

**Feng-Zhen Zhao and Qin Fang**

ON CERTAIN SUMS FOR THE PROBABILITY MASS FUNCTION OF THE  $q$ -DEFORMED BINOMIAL DISTRIBUTION

265-278

**Abstract:** In this paper, certain sums for the probability mass function of the  $q$ -deformed binomial distribution are studied. These sums are related to the  $q$ -analogue of the inverse moments for the zero-truncated positive binomial distributions. For these sums, some recurrence relations are derived and some asymptotic approximations are given. In addition, certain sums involving the probability mass function of the  $q$ -negative binomial distribution are discussed.

**George A. Anastassiou**

APPROXIMATION BY SHIFT INVARIANT MULTIVARIATE SUBLINEAR-SHILKRET  
OPERATORS

279-299

**Abstract:** A very general multivariate positive sublinear Shilkret integral type operator is given through a convolution-like iteration of another multivariate general positive sublinear operator with a multivariate scaling type function. For it sufficient conditions are given for shift invariance, preservation of global smoothness, convergence to the unit with rates. Furthermore, two examples of very general multivariate specialized Shilkret operators are presented fulfilling all the above properties, the higher order of multivariate approximation of these operators is also considered.

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