

# Indian Journal of Mathematics

Volume 45, Number 1, April 2003

---

## CONTENTS

<b>A. Behera and K. K. Dash</b>	
A categorical constructions of a complete metric space	1
<b>A. Behera and K. K. Dash</b>	
Homology decomposition of groups	15
<b>S. Bhargava, chandrashekar Adiga and M. S. Mahadeva Naika</b>	
A new class of modular equations in Ramanujan's alternative theory of elliptic functions of signature 4 and some new $P - Q$ eta-function identities	23
<b>Prem Chandra and Varsha Karanjgaokar</b>	
An aspect of local property of the Fourier series	41
<b>Pratulananda Das</b>	
A note on pseudo-completeness of the density topology on a topological group	53
<b>Raju K. George and Rajesh C. shah</b>	
Solvability and controllability of controlled Lienard system	59
<b>Liu Lanzhe</b>	
Weighted weak type $(H_1, L_1)$ estimates for commutators of Littlewood-Paley operators	71
<b>P. N. Natarajan</b>	
On the algebra $(c_0, c_0)$ of infinite matrices in non-archimedean fields	79
<b>Pramila Srivastava</b>	
Carathéodory outer measure and measurability	89
<b>Stevo Stevic</b>	
On sequences which satisfy a nonlinear inequality	105
<b>V. Yegnanarayanan</b>	
A note on Nordhaus-Gaddum class	117

---

Laser Setting : Jain Lasers, 33 Dilkusha New Katra, Allahabad-211002, U.P., India.  
Printed by : Mudran Kendra, 609-A Subhash Nagar, Allahabad-211002 U.P., India.

## **A CATEGORICAL CONSTRUCTION OF A COMPLETE METRIC SPACE**

A. BEHERA AND K. K. DASH

*(Received 15 January 2001; Revised 9 July 2001)*

Deleanu, Frei and Hilton have developed the notion of generalized Admas completion in a categorical context. The construction of a complete metric space from an incomplete metric space is a well known result in general topology. In this paper, it is shown that this completion of a metric space is, in fact, the Adams completion of the metric space with respect to a suitable set of morphisms in a category.

## **HOMOLOGY DECOMPOSITION OF GROUPS**

A. BEHERA AND K. K. DASH

*(Received 2 March 2001; Revised 27 May 2002)*

Deleanu, Frei and Hilton have developed the notion of generalized Admas completion in a categorical context; they have also suggest the dual notion, namely, the Admas cocompletion of an object in a category. In this paper we obtain the different stages of an ablian group as the cocompletions of the group via homology theory of groups.

## **A NEW CLASS OF MODULAR EQUATIONS IN RAMANUJAN'S ALTERNATIVE THEORY OF ELLIPTIC FUNCTIONS OF SIGNATURE 4 AND SOME NEW $P - Q$ ETA-FUNCTION IDENTITIES**

S. BHARGAVA CHANDRASHEKAR ADIGA AND M. S. MAHADEVA NAIKA

*(Received 2 March 2002; Revised 26 June 2002)*

In this paper we obtain a class of modular equations in Ramanujan's alternative theory of elliptic functions of signature 4 and employ them to obtain a new class of  $P - Q$  eta-function identities with four moduli akin to Ramanujan's.

## **AN ASPECT OF LOCAL PROPERTY OF THE FOURIER SERIES**

PREM CHANDRA AND VARSHA KRANJGAOKAR

*(Received 27 December 1999; Revised 3 January 2001; Re-revised 19 May 2003)*

In this paper a theorem on summability  $|\alpha, \beta|$ ,  $\alpha = 1, \beta > 1$ , of the Fourier series has been proved. The theorem includes the well known result on the local property of  $|C, 1 + b|$  ( $b > 0$ ) due to Bosanquet.

## **A NOTE ON PSEUDI-COMPLETENESS OF THE DENSITY TOPOLOGY ON A TOPOLOGICAL GROUP**

PRATULANANDA DAS

*(Received 24 September 1999; Revised 11 June 2001; Re-Revised 17 July 2001)*

In this short note we investigate when the density topology on a topological group is pseudo-complete.

## **SOLVABILITY AND CONTROLLABILITY OF CONTROLLED LIENARD SYSTEM**

RAJU K. GEORGE AND RAJESH C. SHAH

*(Received 31 August 1999; Revised 23 July 2002)*

In this paper we investigate the solvability and controllability of the Lienard equation. For solvability analysis we use a more general Lienard system which was recently studied by Huang [2] for boundedness of solution. We give sufficient conditions on the nonlinear function in the Lienard system which will ensure the solvability and controllability of the system. In our analysis, we make use of the Bench contraction principle and theory of Lipschitz continuous operators. We also obtain algorithm for the computations of the steering control.

**WEIGHTED WEAK TYPE  $(H^1, L^1)$  ESTIMATES FOR COMMUTATORS OF LITTLEWOOD-PALEY OPERATORS**

LIU LANZHE

(*Received* 11 October 2001; *Revised* 2 April 2002)

We show the weighted weak type  $(H^1, L^1)$  estimates for the commutator of Littlewood-Paley operators.

**ON THE ALGEBRA  $c_0, c_0$  OF INFINITE MATRICES IN NON-ARCHIMEDEAN FIELDS**

P. N. NATARAJAN

(*Received* 22 February 2002; *Revised* 9 September 2002)

In this paper  $K$  denotes a complete, non-trivially valued non-archimedean field. Infinite matrices and sequences have entries in  $K$ . A few results in the context of the algebra  $c_0, c_0$  of infinite matrices under a convolution product are studied.

**CARATHÉODORY OUTER MEASURE AND MEASURABILITY**

P. RAMILAS. RRAMILA

(*Received* 6 March 2003)

Carathéodory's approach is adapted towards generating a measure on a countably complete bounded lattice of function from  $X$  to the closed unit interval  $I$  via an outer measure on  $I^X$ . The conditions satisfied by the generated measure lead to an axiomataic defiof an  $F^*$ -probability measure space is an example.

## ON SEQUENCES WHICH SATISFY A NONLINEAR INEQUALITY

STEVO

(Received 17 October 2001; Revised 23 July 2002)

In this note we prove two generalizations of the following result:

Suppose that  $(a_n)$  and  $(b_n)$  are two sequences of nonnegative numbers such that  $a_{n+1} \leq a_n + b_n$  for all  $n \geq 1$ . If  $\sum b_n < \infty$ , then the sequence  $(a_n)$  converges.

These two results consider sequences of real numbers which satisfy a difference inequality of order equal to  $k = N$ . Also we generalize this result for a system of difference inequalities.

## A NOTE ON NORDHAUS-GSDDUM CLASS

V. YEGNANARAYANAN

(Received 13 March 2001)

The determination of the upper and lower bounds (preferably sharp bounds)  $f(G) + f(G^c)$  and  $f(G)f(G^c)$  where  $G$  is a graph of order  $p$  is called the Nordaus-Gaddum problem for a given graph theoretic parameter  $f$  and a positive integer  $p$ . In this paper we study the sums and products of values of the vertex partition number over the factors of a decomposition.